

Solutions to Exercise # 11

(範圍: Graph Theory)

1. How many different Hamiltonian cycles are there in K_5 ? (10%)

Sol: $\frac{4!}{2} = 12$.

2. Prove the theorem on page 68 of lecture notes. (15%)

Sol: The proof is similar to the proof on pages 65 to 67 of lecture notes. The traversal starts at vertex u , traverses a new edge whenever it is possible, and finally stops at vertex v . If there are untraversed edges, continue the traversal as done at Steps 2 to 4 on pages 66 and 67 of lecture notes.

3. P. 564: 22. (40%)

Sol:

(a) If $x \neq v$ and $y \neq v$, then $\deg(x) + \deg(y) = (n-2) \times 2$.

Otherwise, $\deg(x) + \deg(y) = 2 + (n-2) = n$.

- (b) Yes.

There is a Hamilton cycle in G_n , which can be assured by the theorem on page 79 of lecture notes.

(c) $|E| = \frac{(n-2) \times (n-1)}{2} - 1 + 2 = \frac{(n-2) \times (n-1)}{2} + 1$.

- (d) No.

4. Given a graph G , how to determine 0/1 matrices $B, B^2, B^3, \dots, B^{|V|-1}$ so that for $1 \leq k \leq |V|-1$ and $i \neq j$, $B^k(i, j) = 1$ if and only if there exists an i -to- j walk of length $\leq k$ in G ? (15%)

Sol: Let $B = A + A^0$, where A is the adjacency matrix of G . The addition of A^0 to A is equivalent to adding a loop to each node of G . Thus, an i -to- j walk can increase its length by traversing loops.

5. Given a graph G , how to determine matrices $C, C^2, C^3, \dots, C^{|V|}$ so that for $1 \leq k \leq |V|-1$ and $i \neq j$, $C^k(i, j)$ tells the number of different i -to- j walks of length k in G ?

(20%)

Sol: Let $C=A$, and perform the matrix operation as ordinary mathematical addition and multiplication, e.g., $1 + 1 = 2$ and $2 \times 3 = 6$. When $C^{k-l}(i, r) > 0$ and $C^l(r, j) > 0$ represent the number of different i -to- r walks of length $k-l$ and the number of different r -to- j walks of length l in G , respectively, there are $C^{k-l}(i, r) \times C^l(r, j)$ different i -to- j walks of length k in G , which go via r and have the i -to- r (r -to- j) walks of length $k-l$ (l).