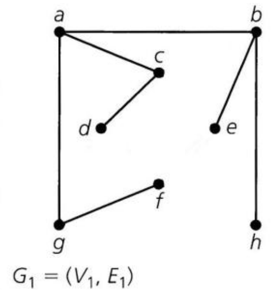
Solutions to Exercise #10

(範圍: Graph Theory)

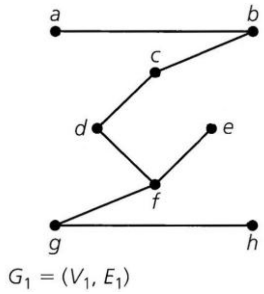
1. Build a BFS spanning tree and a DFS spanning tree of the graph G_1 in Figure 11.42, where it is assumed that vertex a is the root and the priorities of the other vertices to be branched or visited are $b > c > d > e > f > g > h$. (20%)

Sol:

BFS spanning tree:



DFS spanning tree:



2. Suppose that (u, v) is an edge of a graph G and it has the least cost among all edges that are incident to vertex v. Does every minimum spanning tree contain (u, v) (a) when all edges have distinct costs or (b) when multiple edges may have the same cost? Explain your answer. (20%)

Sol:

(a) Yes.

Suppose that T is a minimum spanning tree of G and T does not contain (u, v). If (u, v) is added to T, then a cycle is formed. Assume that (w, v) is the other edge in the cycle that is incident to v. Replacing (w, v) with (u, v) in T will induce a spanning tree whose cost is smaller than T, a contradiction.

(b) No.

If both (w, v) and (u, v) in (a) have the same cost, then T is still a minimum spanning tree of G.

3. Is it possible to obtain a maximum-cost spanning tree of a weighted graph G by modifying Kruskal's algorithm? (10%)

Sol: Yes.

We only need to sort edges of G nonincreasingly according to their costs.

4. Consider the graph of Figure 11.54(a) and assign its edges with costs as follows: $c(a, b) = 62$, $c(a, d) = 37$, $c(a, h) = 45$, $c(b, c) = 19$, $c(b, g) = 28$, $c(c, d) = 70$, $c(c, f) = 53$, $c(d, e) = 81$, $c(e, f) = 15$, $c(e, h) = 40$, $c(f, g) = 39$, and $c(g, h) = 11$. What is the edge sequence obtained by executing Prim's MST algorithm on the weighted graph with starting vertex a? (10%)

Sol: (a, d), (a, h), (g, h), (b, g), (b, c), (f, g), (e, f).

5. For the graph of Figure 12.39(a), first compute $DFN(i)$ and $L(i)$ for each vertex i with the following assumptions when building a DFS spanning tree: vertex c is the root and the priorities of the other vertices to be visited are $a > b > d > e > f > g > h > i$, and then find all articulation points and bridges accordingly. (10%)

Sol:

vertex i :	a	b	c	d	e	f	g	h	i
DFN(i) :	2	3	1	4	5	6	7	8	9
L(i) :	1	1	1	1	1	6	6	6	6

articulation points: c, f.
bridges: (c, f).

6. P. 621: 4 (only for (b)). (10%)

Sol: Suppose that T is a spanning tree of G. There are two or more nodes of T whose degrees are one. According to (a), they are not articulation points of T (and hence G).

7. Let $k_v(G)$ and $k_e(G)$ represent the vertex connectivity and edge connectivity, respectively, of a graph G. (a) Show $k_v(G) \le k_e(G)$. (b) Give an example of $k_v(G) < k_e(G)$. (20%)

Sol:

(a) It suffices to show that for each edge cut E', there exists a vertex cut V' with $|V'| \le |E'|$.

Let $E' = \{(u_1, v_1), (u_2, v_2), \dots, (u_p, v_p)\}$ be an edge cut. Then, the collection of $u_1, u_2, \dots, u_p$ forms a vertex cut V', where $|V'| \le |E'|$.

(b) Refer to the lower 2-connected graph on page 52 of lecture notes. It has $2 = k_v(G) < k_v(G) = 3$.