Solutions to Exercise #6

(範圍: Boolean Algebra, Rings)

1. Let $(K, \cdot, +)$ be a Boolean algebra. A proof of $a \cdot (a+b) = a$ for every $a, b \in K$ was given on page 65 of lecture notes. Please prove $a + (a \cdot b) = a$ for every $a, b \in K$ by the principle of duality. (10%)

Sol: $a + (a \cdot b) = (a + a) \cdot (a + b) = a \cdot (a + b) = (a + 0) \cdot (a + b) = a + 0 \cdot b = a + 0 = a$.

2. P. 741: 4. (20%)

Sol: (a) $x + y = x \cdot y + y = (x + 1) \cdot y = 1 \cdot y = y$.

(b) $x \le y \Rightarrow x + y = y \Rightarrow \overline{x + y} = \overline{y} \Rightarrow \overline{y} \cdot \overline{x} = \overline{y} \Rightarrow \overline{y} \le \overline{x}$.

3. Prove Theorem 14.5 on page 681 of Grimaldi's book. (20%)

Sol: (a) Suppose that u and u' are two unities of R. Then, $u = u \cdot u' = u'$.

(b) Suppose that b and b' are two multiplicative inverses of x, i.e.,

$x \cdot b = b \cdot x = u = b' \cdot x = x \cdot b'$.

Then, $b = b \cdot u = b \cdot (x \cdot b') = (b \cdot x) \cdot b' = u \cdot b' = b'$.

4. P. 678: 2 (only for (b) and (c)). (10%)

Sol: (b) Yes.

(c) Yes.

5. P. 678: 8. (30%)

Sol: (a) x.

(b) The additive inverses of s, t, x and y are t, s, x and y, respectively.

(c) $t \cdot (s + xy) = t \cdot (s + x) = t \cdot s = y$.

(d) Yes (because Table 14.4(b) are symmetric).

(e) No.

(f) (s, y) or (t, y).

6. P. 684: 4. (10%)

Sol: Suppose that a is a unit of R, i.e., $a \cdot b = b \cdot a = u$ for some $b \in R$.
If $a \cdot c = z$, then $b \cdot (a \cdot c) = b \cdot z = z$, which implies $c = z$ because $b \cdot (a \cdot c) = (b \cdot a) \cdot c = u \cdot c = c$.
Therefore, a is not a proper zero divisor of R.