Exercise #10

(範圍: Graph Theory)

1. Build a BFS spanning tree and a DFS spanning tree of the graph G_1 in Figure 11.42, where it is assumed that vertex a is the root and the priorities of the other vertices to be branched or visited are $b > c > d > e > f > g > h$. (20%)

2. Suppose that (u, v) is an edge of a graph G and it has the least cost among all edges that are incident to vertex v. Does every minimum spanning tree contain (u, v) (a) when all edges have distinct costs or (b) when multiple edges may have the same cost? Explain your answer. (20%)

3. Is it possible to obtain a maximum-cost spanning tree of a weighted graph G by modifying Kruskal's algorithm? (10%)

4. Consider the graph of Figure 11.54(a) and assign its edges with costs as follows: $c(a, b) = 62$, $c(a, d) = 37$, $c(a, h) = 45$, $c(b, c) = 19$, $c(b, g) = 28$, $c(c, d) = 70$, $c(c, f) = 53$, $c(d, e) = 81$, $c(e, f) = 15$, $c(e, h) = 40$, $c(f, g) = 39$, and $c(g, h) = 11$. What is the edge sequence obtained by executing Prim's MST algorithm on the weighted graph with starting vertex a? (10%)

5. For the graph of Figure 12.39(a), first compute $DFN(i)$ and $L(i)$ for each vertex i with the following assumptions when building a DFS spanning tree: vertex c is the root and the priorities of the other vertices to be visited are $a > b > d > e > f > g > h > i$, and then find all articulation points and bridges accordingly. (10%)

6. P. 621: 4 (only for (b)). (10%)

7. Let $k_v(G)$ and $k_e(G)$ represent the vertex connectivity and edge connectivity, respectively, of a graph G. (a) Show $k_v(G) \leq k_e(G)$. (b) Give an example of $k_v(G) < k_e(G)$. (20%)