

系所：\_\_\_\_\_ 學號：\_\_\_\_\_ 姓名：\_\_\_\_\_ (4/24)

Suppose that  $(R, +, \cdot)$  is a finite integral domain and  $R = \{d_1, d_2, \dots, d_n\}$ , where  $d_i$ 's are all distinct. Explain why  $R = \{a \cdot d_1, a \cdot d_2, \dots, a \cdot d_n\}$  for each  $a \in R$  and  $a \neq z$ .

Sol. •  $a \cdot d_i \in R$  for  $1 \leq i \leq n$ .

- $a \cdot d_i \neq a \cdot d_j$  for  $i \neq j$ , for otherwise  $d_i = d_j$ , as a consequence of the cancellation law, a contradiction (the cancellation law of multiplication holds for an integral domain).

$$\Rightarrow R = \{a \cdot d_1, a \cdot d_2, \dots, a \cdot d_n\}$$