

系所：_____ 學號：_____ 姓名：_____ (3/20)

Solve $a_n - 4a_{n-1} + 4a_{n-2} = 2^n$, $n \geq 2$, $a_0 = 1$, $a_1 = 2$.

Sol. Let $a_n^h = c \cdot r^n$.

$$r^2 - 4r + 4 = 0.$$

$$\Rightarrow r = 2 \text{ (a root of multiplicity 2).}$$

$$\Rightarrow a_n^h = c_1 2^n + c_2 n 2^n.$$

Let $a_n^p = kn^2 2^n$.

$$kn^2 2^n - 4k(n-1)^2 2^{n-1} + 4k(n-2)^2 2^{n-2} = 2^n.$$

$$\Rightarrow k = 1/2.$$

$$a_n = a_n^h + a_n^p = c_1 2^n + c_2 n 2^n + (1/2) \cdot n^2 2^n.$$

$$a_0 = 1, a_1 = 2 \Rightarrow c_1 = 1, c_2 = (-1/2).$$