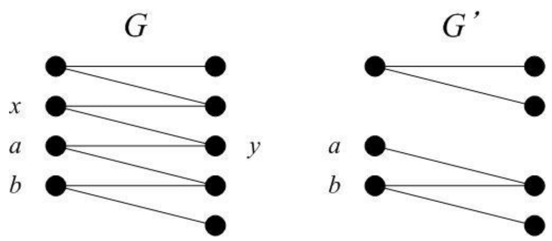
Suppose that $G = (R \cup S, E)$ is a bipartite graph, where $|R| \leq |S|$, and $|W| < |\text{ADJ}(W)|$ for every $W \subseteq R$. The following is a proof (by induction on $|R|$) that G has a complete matching. Please explain why $|W'| \leq |\text{ADJ}(W')|$ in G' holds.

Induction basis. G has a complete matching if $|R| = 1$.

Induction hypothesis. Suppose that G has a complete matching when $|R| \leq m-1$.

Then, consider the situation of $|R| = m$ below.

Arbitrarily pick an edge (x, y), where $x \in R$ and $y \in S$.
Let G' be the graph obtained from G by removing x, y.



For every $W' \subseteq R - \{x\}$,

$|W'| < |\text{ADJ}(W')|$ in G

$\Rightarrow$ $|W'| \leq |\text{ADJ}(W')|$ in G'

$\Rightarrow$ G' has a complete matching

Sol. $|\text{ADJ}(W')|$ in G' = $|\text{ADJ}(W')|$ in G or $(|\text{ADJ}(W')|$ in $G) - 1$.

$|\text{ADJ}(W')|$ in G' = $|\text{ADJ}(W')|$ in G or
$|\text{ADJ}(W')|$ in G' = $(|\text{ADJ}(W')|$ in $G) - 1$.

(The latter holds when $y \in \text{ADJ}(W')$ in G).

Hence,

$|W'| < |\text{ADJ}(W')|$ in G (the assumption of this case)

$\Rightarrow$ $|W'| \leq |\text{ADJ}(W')|$ in G'

$\Rightarrow$ G' has a complete matching
(by induction hypothesis)